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HEAT TRANSFER IN A CHANNEL WITH AN UNHEATED LENGTH

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Аннотация—Исследована теплоотдача в прямоугольном канале при помощи встроенного альфакалориметра. Полученные данные свидетельствуют о том, что в условиях внутренней задачи предварительная гидродинамическая стабилизация потока не влияет на теплообмен.

NOMENCLATURE

$t(\tau)$, temperature of the core;
$t_1(z, r, \tau)$, temperature of plexiglass;
$t_2(r, \tau)$, temperature of cellular resin;
T, like $t(\tau)$ at stationary conditions;
$T_1(z, r)$, like $t_1(z, r, \tau)$ at stationary conditions;
$T_2(r)$, like $t_2(r, \tau)$ at stationary conditions;
t_0, temperature of the flow;
t_w, temperature of the slab;
ϑ, surplus temperature of the core;
G, weight of the core;
c_p, heat capacity of the core;
λ_1, thermal conductivity of plexiglass;
λ_2, thermal conductivity of cellular resin;
a_1, thermal diffusivity of plexiglass;
a_2, thermal diffusivity of cellular resin;
r_1, radius of the cylindrical part of the core and spherical surface;
r_2, radius of the hole in the slab, radius of the hemispherical recess in the cover of the calorimeter, $f = \pi r_1^2$, $f_1 = 2\pi r_1 l$, $f_2 = 2\pi r_1^2$;
α_0, heat-transfer coefficient in a duct with a thermally and hydrodynamically developed flow;
m, rate of the regular cooling;
W, power of the heater;
l, height of the cylindrical part of the core.

INTRODUCTION

Ambrazyavichus and Zhukauskas [1] studied heat transfer from a surface in a longitudinal flow and correlated their data as Nu versus Re, both numbers related to the length of the heated section. The authors used the latter as a characteristic dimension and this allowed them to correlate the data by the relation identical with the conventional one used for calculation of the natural flow round a plate without an unheated starting length, i.e. when the flow enters the heated section being hydrodynamically undeveloped. Mikheev [2] is also plussed about the length of a heated section as a characteristic dimension.

It is natural to obtain the Nusselt number based on the length of a heat-transfer surface that corresponds to a physical meaning of this number, as it was stated in [3]. However, the number Re based on a partial length of the surface does not characterize the conditions of the flow. For practical convenience different characteristic dimensions should be therefore adopted for Nu and Re, the size factor l_e/l being introduced into the characteristic equation. This is better grounded theoretically. The corresponding calculation formula is obtained by means of formal transformation of the conventional relation

$$Nu_e = aRe_e^{0\cdot 8}\,\varphi(Pr),\; Nu_e = \frac{\alpha l_e}{\lambda},\; Re = \frac{wl_e}{\nu} \quad (1)$$

to such a form:

$$Nu_e = aRe^{0\cdot 8}\,\varphi(Pr)\left(\frac{l_e}{l}\right)^{0\cdot 8},\; Re = \frac{wl}{\nu}. \quad (2)$$

Chaplina, [3], used equation (2) for correlation of her data. Her experiments confirm once more Mikheev's conclusion on independence of heat transfer of the unheated length.

Consider the effect of hydrodynamic pre-history of the flow on heat transfer for the internal problem. This problem is of great interest at present since recently the methods for measuring heat transfer by an instrument which may be referred to as inserted alphacalorimeter have been widely adopted. In the wall of a channel, a small plate, thermally insulated from it, is inserted flush with the flowed surface. The temperature gradient arises in the plate caused by intense cooling. The heat flow and heat-transfer coefficient are found from this gradient. Other methods of measuring the heat flow are also known. An inserted alphacalorimeter was used, for instance, for study of convective heat transfer in combustion chambers of gas turbines [4, 5, 6, 7], in small-sized furnace chambers of high intensity [8, 9], etc.

In all the above works heat-transfer coefficients have the meaning of heat-transfer rate characteristics which correspond to thermally and hydrodynamically developed flow.

1. EXPERIMENTAL APPARATUS

The test section of the unit is a thermally insulated rectangular duct of section 200 × 85 mm and 1600 mm long made of steel polished plates. Into the duct an iron 200 × 45 mm plate is inserted which forms two similar slot channels of section 200 × 20 mm. Their width may be diminished to 10,5 and 3 mm by means of making-up strips. Entry lengths of channels were profiled as lemniscate. The duct was connected with an elevated tank, 700 mm in diameter, provided with a damping screen.

The air velocity was measured by a standard diaphragm placed before the elevated tank. A uniform air flow in the channels was controlled by pressure nozzles.

Heat transfer in the channels was measured by two alphacalorimeters imbedded in the plate and fixed at 1200 and 1375 mm from the entry section. A schematic drawing of the alpha-calorimeter is shown in Fig. 1. The calorimeter is composed of a cylindrical duralumin core (1) with flat and spherical end surfaces. On the cylindrical part of the core a plexiglass ring (2) was mounted. The space between the spherical end surface of the core and the alumin cover (3) was filled with cellular resin which is an excellent heat insulating material. On the spherical end-surface of the core two grooves were made. In one of them a chromelalumel thermocouple 0·2 mm in diameter was inserted. A heater constructed of a nickel–chrome wire 0·2 mm in diameter was let into the other groove. The alphacalorimeters were inserted into cylindrical holes made in the slab with a tightness of 0·1 mm which provided good thermal contact between the core and the slab and between the cover and the slab. The cover was made of the material of high thermal conductivity which allowed the temperature of the external surface of the insulation (plexiglass and cellular resin) to be maintained constant and equal to the temperature of the slab. The temperature in the latter was actually uniform which was provided by the flow over both surfaces of the slab and by adequate heat insulation of the whole test section. Temperature distribution over the length and the cross-section of the slab were measured by ten thermocouples.

2. EXPERIMENTAL PROCEDURE AND DISCUSSION OF THE DATA

The heat-transfer coefficient was determined by the methods of the regular thermal regime and by those of the stationary heat flow. In the first case the core was heated to a temperature 10–15°C higher than that of the flow, and then the cooling rate was measured with the electrical heater disengaged. In the second case (the methods of a stationary heat flow) measurements were carried out with the constant power of the heater which was controlled so that the surplus temperature of the core was about 10°C.

The heat-transfer coefficient was found from the heat balance equation for the core of the calorimeter:

$$W - c_p G \frac{\mathrm{d}t}{\mathrm{d}\tau} = \alpha (t - t_0) f$$

$$- \lambda_1 \int_{f_1} \frac{\partial t_1}{\partial r}\bigg|_{r=r_1} \mathrm{d}f_1 - \lambda_2 \int_{f_2} \frac{\partial t_2}{\partial r}\bigg|_{r=r_1} \mathrm{d}f_2. \quad (1)$$

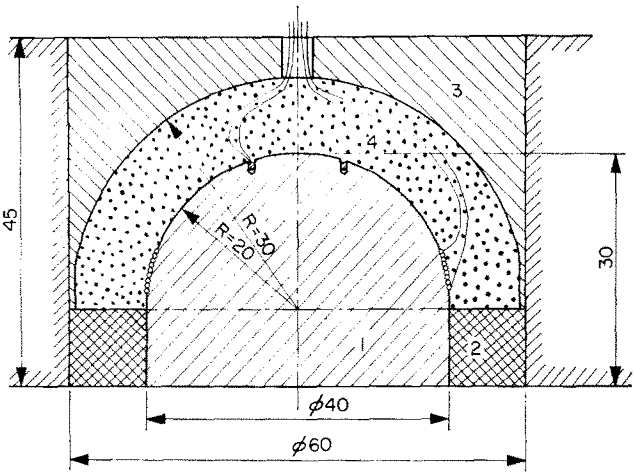

FIG. 1. Schematic design of the alphacalorimeter: 1—core of the alphacalorimeter; 2—plexiglass insulation; 3—cover; 4—cellular resin insulation.

It was solved for the regular regime of cooling at $W = 0$ and at $W = \text{const.}$ for the stationary heat transfer. The temperature field of the core was assumed uniform.

The temperature distribution in the plexiglass insulation is governed by the system of equations:

$$a_1\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t_1}{\partial r}\right)+\frac{\partial^2 t_1}{\partial z^2}\right]=\frac{\partial t_1}{\partial \tau},\quad r_1 \leqslant r \leqslant r_2$$

$$r = r_1,\ t_1 = t;\ r = r_2,\ t_1 = t_w = \text{const.} \quad (2)$$

$$z = 0,\ \frac{\partial t_1}{\partial z} = 0;\ z = l,\quad \alpha(t_1 - t_0) = -\lambda_1\frac{\partial t_1}{\partial z}. \quad (3)$$

For hollow hemispheres we have:

$$\frac{a_2}{r}\frac{\partial^2}{\partial r^2}(rt_2) = \frac{\partial t_2}{\partial \tau},\quad r_1 \leqslant r \leqslant r_2$$

$$r = r_1,\ t_2 = t,\ r = r_2,\ t_2 = t_w = \text{const.} \quad (4)$$

In (3) and (4) absence of heat transfer between the layers of thermal insulation is assumed at $z = 0$. This is based on the assumption that in the region where the side and the face layers of insulation are in contact, the heat flow is directed radially from the core to the slab.

At the prescribed boundary conditions the stationary temperature field is obviously non-uniform. For the stationary conditions the heat conduction equations will be written as follows:

$$\alpha(T - t_0)f = \lambda_1\int_{f_1}\left.\frac{\partial T_1}{\partial r}\right|_{r=r_1} df_1 + \lambda_2\int_{f_2}\left.\frac{dT_2}{dr}\right|_{r=r_1} df_2 \quad (5)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right)+\frac{\partial^2 T_1}{\partial z^2} = 0$$

$$r = r_1,\ T_1 = T;\ r = r_2,\ T_1 = t_w = \text{const.}$$

$$z = 0,\ \frac{\partial T_1}{\partial z} = 0;\ z = l,\ \alpha(T_1 - t_0) = -\lambda_1\frac{\partial T_1}{\partial z} \quad (6)$$

$$\frac{d^2}{dr^2}(rT_2) = 0,\ r = r_1,$$

$$T_2 = T;\ r = r_2,\ T_2 = T_w = \text{const.} \quad (7)$$

According to the generalized Kondrat'ev's theory of the regular thermal regime [10] the system at non-uniform boundary conditions is controlled by the surplus temperature related to that of the stationary conditions, i.e.

$$t - T = A\exp(-m\tau),$$
$$t_1 - T_1 = A_1 u\exp(-m\tau),$$
$$t_2 - T_2 = A_2 v\exp(-m\tau). \quad (8)$$

It follows from equations (1–8) that eigenfunctions u and v of the boundary problem should satisfy the following equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial z^2}+\frac{m}{a_2}u=0$$

$$z=0,\ \frac{\partial u}{\partial z}=0;\ z=l,\ au+\lambda_1\frac{\partial u}{\partial z}=0,$$

$$r=r_1,\ u=1;\ r=r_2,\ u=0 \qquad (9)$$

$$\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}(rv)+\frac{m}{a_2}v=0,\ r=r_1,$$

$$v=1;\quad r=r_2,\quad v=0 \qquad (10)$$

and the desired relation between the cooling rate and the heat-transfer coefficient is

$$\alpha=m\frac{c_pG}{f}+\frac{\lambda_1}{f}\int_{f_1}\frac{\partial u}{\partial r}\bigg|_{r=r_1}\mathrm{d}f_1+\frac{\lambda_2}{f}\int_{f_2}\frac{\mathrm{d}v}{\mathrm{d}r}\bigg|_{r=r_1}\mathrm{d}f_2. \qquad (11)$$

The derivatives u and v will be determined from (9) and (10).

Let us use the Fourier finite cosine transformation

$$\bar{u}_K=\int_0^l u\cos p_K z\,\mathrm{d}z$$

where p_K are the roots of the equation

$$p_K\tan p_K l=\frac{\alpha}{\lambda_1}.$$

For the image of the function u we find the equation

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\bar{u}_K}{\mathrm{d}r}\right)+\nu_K^2\bar{u}_K=0,\quad \nu_K^2=\frac{m}{a}-p_K^2. \qquad (12)$$

For the prescribed boundary conditions equation (12) will be solved as follows:

$$\bar{u}_K=\frac{\sin p_K l}{p_K}\frac{Z_0(\nu_K r)}{Z_0(\nu_K r_1)}$$

where

$$z_0(\nu_K r)=J_0(\nu_K r)Y_0(\nu_K r_2)-J_0(\nu_K r_2)Y_0(\nu_K r).$$

By passing to the original we obtain

$$u=\sum_{K=1}^{\infty}\frac{4\sin p_K l}{2p_K l+\sin 2p_K l}\frac{Z_0(\nu_K r)}{Z_0(\nu_K r_1)}\cos p_K z. \qquad (13)$$

The solution of the system of equations (9) is of the following form

$$v=\frac{r_1}{r}\frac{\sin(r-r_2)n}{\sin(r_1-r_2)n},\quad n^2=\frac{m}{a_2}. \qquad (14)$$

Substituting (13) and (14) into (10), we finally obtain

$$\alpha=m\frac{c_pG}{cf}+\frac{\lambda_1 f_1}{lf}\times\sum_{K=1}^{\infty}\frac{4p_K\sin^2 p_K l}{2p_K l+\sin 2p_K l}\frac{Z_0^1(\nu_K r_1)}{p_K Z_0(\nu_K r_1)}-\frac{\lambda_2 f_2}{f}\left[n\cot(r_2-r_1)n+\frac{1}{r_1}\right]. \qquad (15)$$

Relation $\alpha=F(m)$ was plotted beforehand according to (15). The physical parameters of the core and the Plexiglass entering the equation were measured at the Laboratory of Thermal Measurements of the Leningrad Institute of Precise Mechanics and Optics; for cellular resin the manufacturer's data were used.

The experimental data were finally correlated in such a simple way: the heat-transfer coefficient was obtained from the plot $\alpha=F(m)$ according to the rate of cooling measured experimentally. The cooling rate of the core of the calorimeter was found by plotting $\ln(t-T),\tau$.

The method of generalized thermal regime was adopted by reason of the slight difference between the temperatures of the plate and of the flow while the experiments duration was relatively short.

When the heat-transfer rate was measured by the stationary heat flow method after the unit was heated by an air flow, the temperature of the core was preliminary measured. Then the electrical heater was engaged and the stationary regime was established. Because of the low heat capacity of the system the stationary thermal conditions of the core were obtained for 20–30 min. The heat-transfer coefficient was calculated from the formula:

$$\alpha=\frac{W}{f\vartheta}+\frac{\lambda_1 f_1}{lf}\sum_{K=1}^{\infty}\frac{4\sin^2 p_K l}{2p_K l+\sin 2p_K l}\frac{W_0'(p_K r_1)}{p_K W_0(p_K r_1)}-\frac{\lambda_2 f_2}{r_1 f_1}\frac{r_2}{r_2-r_1}$$

$$W_0(p_K r)=I_0(p_K r)K_0(p_K r_2)-I_0(p_K r_2)K_0(p_K r). \qquad (16)$$

Formula (16) was obtained by solution of the system of equations (1–4) at $\tau \to \infty$, $W = \text{const.}$ The solution is reduced to the above considered if the surplus temperature related to that of the stationary thermal regime is introduced at the heat source switched out.

The results of the experiments with slot channels 3, 5, 10 and 20 mm wide are shown in Fig. 2. The experimental points are plotted using the data obtained by both methods. To get reliable data, two calorimeters were used for each method.

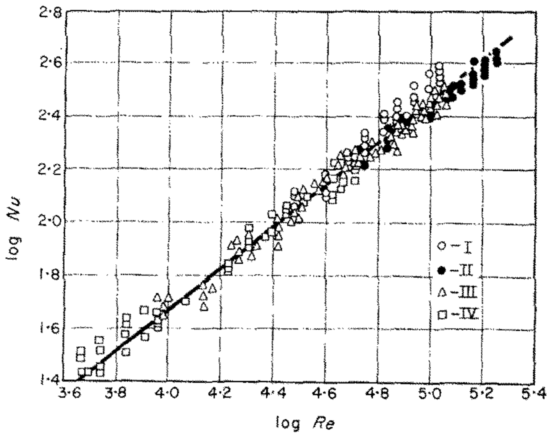

FIG. 2. Effect of Re on Nu according to the data obtained by measuring by alphacalorimeters. Width of the slot: I—3 mm; II—5 mm; III—10 mm; IV—20 mm.

With the channels of the above width the calorimeters were placed at $L/d_{eq} = 30$–230 mm from the entry length, i.e. if the flow is fully or partially hydrodynamically developed.

Let hydrodynamic prehistory of the flow have no effect on heat transfer for the internal problem. Then heat transfer of the calorimeters will be the same as that in the entry length. At $L/d_{eq} < 10$ heat transfer may be calculated by the formulae for a plate if Re is found by a maximum air velocity along the centre-line of the channel. Experimental data in Fig. 2 are correlated by this method. The Nusselt and Reynolds numbers are based on the length equivalent of the flow round the core ($l_{eq} = 35{\cdot}3$ mm). The curve in Fig. 2 is well described by the equation

$$Nu = 0{\cdot}029\, Re^{0{\cdot}8}. \tag{17}$$

The coefficient in equation (17) is 10 per cent lower than that in Mikheev's formula [2], 3 per cent higher than that in Jacob and Dow's work [11] and it is actually the same as that obtained by Chaplina [3]. Thus, the assumption that hydrodynamic prehistory of the flow for the internal problem does not effect heat transfer is confirmed and the consequence is the following: heat transfer in ducts with the unheated starting length may be calculated from the conventional formula for the total heated length; in this case a parameter based on L measured from the section where heating begins, accounts for L/d. In view of the above said, one can see that the recommendations given in [4–9] are unfounded because they are based on the assumption that alphacalorimeter measures the heat transfer coefficient corresponding to the flow thermally and hydrodynamically developed along the whole length of the channel. Actually, since the temperature changes of the wall of the calorimeter are jumplike, the conditions of the flow round the calorimeter are such as though the flow is thermally undeveloped. For the channel of interest when the flow is thermally and hydrodynamically developed

$$Nu_0 = 0{\cdot}018\, Re_0^{0{\cdot}8}. \tag{18}$$

Dividing (17) by (18), we obtain

$$\frac{\alpha}{\alpha_0} = 1{\cdot}88 \left(\frac{d_{eq}}{l_{eq}}\right)^{0{\cdot}2}.$$

If the diameter equivalent of the channel d_{eq} is equal to the length equivalent of the calorimeter, then the heat-transfer coefficient measured by the calorimeter is 88 per cent larger than the actual value for the developed flow.

It should be noted that use of the diameter equivalent of the fire tube of the combustion chamber in [4–7] or of the furnace in [8, 9] as a characteristic dimension for Nu is unjustified since this contradicts the physical meaning of the number which is actually determined for the calorimeter.

Dwell upon the experimental data by Narezhny [4, 5]. The conditions of his experiments differ from those described above only by the temperature of the flow and that of the wall. The calorimeters were mounted on the thermally insulated wall with the temperature close to that of the flow. Several calorimeters were inserted along the chamber.

As is shown in [4] equal rate of heat transfer was found in all the sections along the chamber. It follows from this that the heat-transfer coefficient is constant on the wall.

In view of the above said calorimeters of equal size placed in flow of the same velocity should measure equal average values of heat-transfer coefficients. However, it does not mean that the heat-transfer coefficient is a constant value over the whole length of the channel of interest. On the contrary, in short ferrules of fire tubes of combustion chambers, essential changes of the heat-transfer coefficient over the length is expected since the flow is thermally undeveloped.

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Abstract—A study is made of heat transfer in a rectangular duct by embedded alpha-calorimeters. The results indicate that under the prevailing conditions preliminary hydrodynamic stabilization of the flow does not influence the heat exchange.

Zusammenfassung—Mit Hilfe eines eingebauten α-Kalorimeters wird der Wärmeübergang in einem rechteckigen Kanal untersucht. Im Falle des inneren Problems zeigt sich, dass eine hydrodynamische Anlaufstrecke keinen Einfluss auf den Wärmeübergang hat.